Direct Method of Parameter Estimation for Steady State Flow in Heterogeneous Aquifers with Unknown Boundary Conditions

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ABSTRACT

In calibrating groundwater flow models, existing parameter estimation techniques typically assume that model boundary conditions (BC) are either known or can be determined from model calibration. However, due to limited subsurface access, BC are rarely known at most field sites, while BC calibration may lead to non-uniqueness in the estimated parameters, BC, and flow field. This could mean that multiple sets of calibrated parameters and BCs can satisfy the model calibration criteria (i.e., minimization of an objective function), which may lead to a wrong model being used for prediction and management purposes. In this work, a new groundwater parameter estimation technique is introduced, which allows the simultaneous estimation of model parameters, model source/sink, and the unknown model boundary conditions.

INTRODUCTION

In Irsa & Zhang (2012), a new direct inversion method was developed to simultaneously estimate steady-state hydraulic conductivities (K), state variables, and BC for a confined aquifer. Unlike the objective-function-based parameter estimation techniques, this method does not require forward groundwater flow simulations to assess the data-model misfits, thus the knowledge of BC is not needed. Instead, it directly incorporates noisy observed data (hydraulic heads and flow rates) at the measurement points in a single step, without solving a boundary value problem. For problems where the underlying conductivity zonation is unknown, the method yields equivalent conductivities without conducting pumping tests. Given sufficient measurement data, the method yields well-posed systems of equations that can be solved efficiently with linear optimization. The solution is also stable when measurement errors are increased. In Irsa & Zhang (2012), the technique was tested on two-dimensional groundwater flow problems with regular and irregular geometries, different heterogeneity patterns, variances of heterogeneity, and error magnitudes. In all cases, parameters (hydraulic conductivities) converge to the correct or expected values and are thus unique, based on which heads and flow fields are constructed directly via a set of analytical expressions. Accurate boundary conditions are then inferred from these fields. For a hydrofacies models with multiple conductivity zones, however, only one hydraulic conductivity can be estimated (the other conductivities are obtained from known conductivity ratios in the form of prior information equations), while subsurface flow rates, rather than velocity or flux observations, are used as measurements. Problems with pumping wells are not addressed.

PROBLEM DESCRIPTION

The method of Irsa & Zhang (2012) has been extended to two-dimensional confined aquifers with wells, to three-dimensional confined problems, and to unconfined aquifers where the flow equation is nonlinear and where the unknown water table is also the unknown BC. In this work, to resolve the strong nonlinearity in unconfined systems where the aquifer is also subject to significant source/sink effects (e.g., areal recharge, pumping discharge), nonlinear optimization techniques have been adopted. These techniques allow the removal of the conductivity ratio constraints, therefore multiple conductivities and multiple recharge (or discharge) rates can be simultaneously estimated along with the unknown model BC. In cases where the heterogeneity patterns (both of K and recharge rates) are unknown, equivalent conductivities and average recharge rates can be estimated. Under the Dupuit Forchheimer assumption of negligible vertical flow, inversion of unconfined aquifers has been successfully accomplished in both one- and two-dimensions. Furthermore, flow rate measurements are not required for the inversion to succeed, where problems have been successfully inverted using hydraulic heads and as few as one